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Arithmetic Reasoning

Strategies and Tactics

As you review the calculations, formulas and problems presented note the similarities with their resolutions. The steps that guide solution thinking are remarkably similar for simple and complex problems. With minor knowledge of formulas, due diligence in reading the problems and practice, you will find you can solve many simple problems quickly and with ease. Once you have the basics down, you will see that solving more complex problems use the same processes and techniques.

This section addresses some of the subtle points that you may find worth reviewing as a reminder before the exam.

1. Read the problem carefully. Make sure your answer is in the units the question is asking for. It is critical to determine the measure and the unit being requested.

Examples

<table>
<thead>
<tr>
<th>Question</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the speed of the car in feet/second?</td>
<td>Not MPH but feet/second</td>
</tr>
<tr>
<td>What is her height in meters?</td>
<td>The answer should be in meters</td>
</tr>
<tr>
<td>What is the length of the fence?</td>
<td>This is focused on perimeter in feet not the area (square feet)</td>
</tr>
</tbody>
</table>

1. Read the problem carefully. Make sure your answer is in the units the question is asking for. It is critical to determine the measure and the unit being requested.
2. Arithmetic is a logical process that requires visualization to mentally process solutions.
   • Think through what is really being asked in the question
   • Determine what steps are required to solve the problem
   • Diagram or chart the information.

3. Organize your thoughts so you can visualize a path to the solution.
   • Think over what calculations are required
   • Break the calculation down into parts and logically sequence them

4. Use scrap paper to organize the calculations so you can review if necessary.
   • Make sure you have adequate space on your scrap paper
   • Track each step of the calculation (this will make double checking your work easier too)

5. Be cautious about the units of measure throughout the process.
   • Label all numbers with appropriate units of measure
   • Examples:
     2ft x 3ft = 6 sq ft
     3in x 2in x 2in = 12 cu. in.
     speed = miles per hour
     speed = feet per second
     .50 = 50%

The problem categories found in the section include:

- Ratios
- Distance / Velocity
- Space Relationships
- Counting - Segregation
- Ratio / Proportion
- Temperature
- Work / Time
Contents

Rational Numbers
  Integers
  Fractions

Decimals

Percentages

Averages / Means and Medians
Arithmetic Calculations

This section provides an overview of the basic arithmetic concepts and calculations found in problems associated with organizational practices in New York State. While most of the concepts will be familiar, some of them will be new or are items you have not seen for a long time. This guide is meant as a refresher and tool to support the mental processes used in arithmetic problem solving. This information has been condensed to closely resemble the language used on the Civil Service Test. Additional practice questions are provided in a supplement to this guide.

Rational Numbers:
The set of integers {...-4, -3, -2, -1, 0, 1, 2, 3, 4…} and fractions {a/b, ½, 3/16} are called rational numbers. These represent the foundation of all the calculations included in this guide.

Integers:
DEFINITIONS

Positive Integers 0, 1, 2, 3, 4…
Negative Integers -1, -2, -3, -4…
Even numbers are divisible by 2

Integers can be:

- Added (1) + (-2) + (2)= 1
- Subtracted (2) - (12) = -10
- Multiplied (-2) x (4) = -8
- Divided (10) ÷ (2) = 5

Calculations:
Product of Prime Factors:

Various combinations of numbers have the same product when multiplied.

- 24 = 2 x 12
  = 4 x 6
  = 2 x 2 x 2 x 3

- 252 = 2 x 126
  = 2 x 2 x 63

QUIZ (A):
A group of 45 managers at a conference sit in rows with the same number of managers in each row. Each row must contain 2 managers and there must be at least 2 rows. A row is parallel to the front of the room.

How many different arrangements are possible?
Least Common Multiples (LCM):
This is the smallest whole number that is **divisible by each** of the numbers.
- The LCM of 11 and 12 is 132
- The LCM of 8 and 6 is 24

**QUIZ (B):**
Find the LCM of 5, 6 and 15.

**QUIZ (C):**
Find the LCM of 11 and 12.

**Fractions:**
A fraction represents a portion of a whole and is comprised of two integers (i.e., \(\frac{1}{2}\) of 1 is .5 or 50% of the whole).

**DEFINITIONS**
The top number is the numerator (the part) and the bottom number denominator (the whole).
- \(\frac{3}{7}\) as it applies to a piece of pie means that we are referring to 3 pieces from total of 7 pieces cut.

**RULES**
- A denominator of 1 means the fraction is equal to the numerator. For example, \(\frac{12}{1} = 12\)
- When the numerator and denominator are equal, the fraction equals 1. For example, \(\frac{12}{12} = 1, \frac{3}{3} = 1\), etc.
- Convert fractions to the lowest terms. For example, \(\frac{3}{6}\) should be written as \(\frac{1}{2}\) and \(\frac{4}{12}\) should be reduced to \(\frac{1}{3}\).
- A mixed number is a whole number followed by a fraction. (i.e., \(6 \frac{3}{8}\) or \(4 \frac{1}{7}\))

To convert a mixed number to a fraction, multiply the whole number by the denominator of the fraction, and add the numerator to find the numerator of the improper fraction. Then display this number over the original denominator of the improper fraction.

**QUIZ (D):**
Write \(4 \frac{1}{7}\) as a fraction.
Adding / Subtracting Fractions:
Change to equivalent fractions with a common denominator (convert the denominators to the same integer) then add numerators using the least common denominator found. For subtraction we do the same conversion to a common denominator, then subtract the numerators.

Remember that the least common denominator is found by determining the smallest number that can be divided evenly (no remainder) by all the numbers in the denominators and then multiplying the numerators and denominators by the same number so the fraction value has not changed (but the common denominator has been found).

Example:
What is \( \frac{1}{2} + \frac{3}{4} \)?

First, convert \( \frac{1}{2} \) to \( \frac{2}{4} \) by finding the LCM of the denominators:

\[
2 = 2, 4, 6, \ldots
\]
\[
4 = 4, 8, 12, \ldots \quad \text{So the LCM is 4.}
\]

Then, convert \( \frac{1}{2} \) to a fraction with a denominator of 4. A simple method is to say “what multiplied by 2 equals 4?” \( 2 \times 2 = 4 \). Then multiply 2 by the numerator and the denominator of the fraction \( \frac{1}{2} \). \( 1 \times 2 = 2 \) and \( 2 \times 2 = 4 \). Therefore, \( \frac{1}{2} = \frac{2}{4} \).

Now we have both fractions converted using a denominator of 4 (remember that \( \frac{3}{4} \) did not need converting). All we have to do now is add the numerators. For subtraction, simply subtract the numerators.

\[
\frac{2}{4} + \frac{3}{4} = \frac{5}{4} \text{ or } 1 \frac{1}{4} \quad \quad \frac{3}{4} - \frac{2}{4} = \frac{1}{4}
\]

QUIZ (E):
Add the measurements of:
1/2” + 3/8” + 1/4”
Multiplying Fractions:
Multiplying fractions is simply multiplying numerators and denominators. Common denominators are not needed.

Example:
\[
\begin{align*}
1/3 \times 1/4 &= 1/12 \text{ (1x1 over 3x4)} \\
3/4 \times 2/5 &= 6/20 = 3/10 \text{ (3x2 and 4x5)}
\end{align*}
\]

QUIZ (F):
Our team saved 1/4 of $240 raised in the fundraiser. What is the amount saved?

Dividing Fractions:
Dividing one fraction by another requires you to invert the divisor (the reciprocal) and then multiply the first and second fractions. The reciprocal is found by flipping the second fraction. Common denominators are not needed.

Example:
\[
\begin{align*}
3/5 \div 1/4 &= \\
3/5 \times 4/1 &= 12/5 = 2 \frac{2}{5}
\end{align*}
\]

QUIZ (G): A supervisor can produce a report in 2/3 of an hour. How many reports can he produce in 4 hours?

QUIZ (H): It takes 1 1/4 hours to get from Syracuse to Utica traveling at a constant rate of speed. What part of the distance is traveled in 3/4 of an hour?
Decimals:
A decimal is a series of integers after a decimal point. Each decimal represents a fraction over powers of 10. The first three digits to the right of the decimal point are the tenths position, hundredths position and thousandths position respectively.

Example:
\[
\begin{align*}
.5 \text{ (five tenths)} &= 5/10 = 1/2 \\
.32 \text{ (32 hundredths)} &= 32/100 \\
.7 \text{ (seven tenths)} &= 7/10
\end{align*}
\]

Each integer after the decimal indicates the value of the denominator when converting to a fraction (each place after the decimal that is not a zero is equal to the number of zeros in the denominator remembering that it is to a power of 10).

Example:
\[
\begin{align*}
xyz &= xyz/1000 \\
xy &= xy/100 \\
x &= x/10
\end{align*}
\]

Adding / Subtracting Decimals:
Align the decimal points (so digits of each place value are under each other) and add the whole numbers vertically.

Example:
\[
\begin{array}{c}
5.3 \\
100.025 \\
+ 39.61 \\
\hline
8.2 \\
153.135
\end{array}
\]

The subtraction process is merely first aligning the decimals then subtracting the integers.

Example:
\[
\begin{array}{c}
501.30 \\
- 86.15 \\
\hline
415.15
\end{array}
\]

QUIZ (I):
Find the fraction that number .6541 represents.

QUIZ (J):
In collecting change for a fundraising drive the 3 locations have $100.27, $48.90 and $1000.05. How much has been collected?

QUIZ (K):
Of the $1149.22 collected 20% or 1/5 is to be allocated for expenses. How much remains in the account after expenses are taken?
Multiplying / Dividing Decimals:
This process is identical to regular multiplication and division of integers only the answer must have the same number of digits behind the decimal point as there are digits behind the decimal points in all the factors.

Example:

\[
\begin{align*}
5.07 & \times 0.09 \\
& = 0.4563
\end{align*}
\]

Note there are 4 digits behind the decimal points (2 in the first factor and two in the second)

Dividing decimals requires the decimal point in the divisor (denominator) to be moved behind the last digit. Correspondingly, the decimal point in the dividend (numerator) has to be moved the same number of places.

Example:

\[
\begin{align*}
3.50 & \div 0.05 \\
& = 70
\end{align*}
\]

Because there are 2 places after the decimal point for .05 you must move the decimal point over two places in the number 3.50 as well

QUIZ (L):
If travel is reimbursed at .485 cents per mile, what is the cost for 60 miles?

QUIZ (M):
If apples cost $.42 each how many apples can you buy for $5.04?
Percentage:
Percentage is a way of expressing a fraction based in terms of one hundred.

Examples:
1/2 = 50/100 = 50%
3/10 = 30/100 = 30%
.562 = 56.2%
.204 = 20.4%

Some examples of fractions to percentages are as follow:
1% = 1/100
10% = 10/100 = 1/10
90% = 90/100 = 9/10
100% =1/1= 1
120% = 120/100 = 12/10 = 6/5
150% = 150/100 = 15/10 = 3/2

Interest / Discount:
Interest and discount are the most popular uses of percentage calculations. There are two formulas that guide these.

Interest Formula:
Interest = Amount x Time x Rate

Interest
In this formula, the amount is the principal amount, which is the amount borrowed or lent. The time is expressed in the same period as the rate.
For example, you borrow $300 for 2 years at a rate of 10% simple interest per year. 300 x 2 x .1 = 60. The interest you will pay is $60.00.

Simple versus Compound Interest
Simple interest is based on interest during one year. Compound is calculated based on multiple years where interest and principle is added at the end of one year and then multiplied by interest percentage again in the next year, therefore adding more interest over multiple years.

QUIZ (N):
The population of government workers in the central region was 10,000 in 2000. It is projected to increase by 12% by 2015. What is the projected population?
Simple Interest
Carolyn borrowed $5000 from the bank at 6% simple interest per year. If she borrowed money for two years, find the total amount she paid the bank.

Interest = $5000 x 2 x .06 = $600
$5000+$600= $5600 total paid to the bank

Compound Interest
What will $1000 be worth after 3 years if it earns rate of 5% compounded annually?

Year 1 interest is $1000 x 1 x 5% = $50
$1050 is the total amount at the end of year 2

Year 2 interest is $1050 x 1 x 5% = $52.50
$1102.50 is the total amount at the end of year 2

Year 3 interest is $1102.5 x 1 x 5% = $55.13
$1157.63 is the total amount at the end of year 2

$1157.63 is the amount the initial investment of $1000 will be worth in 3 years.

Discounts:
Discounts are a reduction in price.

Discount Formula:
Discount = Cost x Rate of Discount

Discount Rate Formula:
Rate of Discount = \frac{\text{Cost} - \text{Price}}{\text{Cost}} (this equals the amount of the discount)

Example:
What is the discount on a $1500 computer if the discount rate is 10% for employees?

1500
x .10
$150 is the discount.

Conversely, you can calculate the rate of the discount:

Example:
What is the rate of discount if a laptop which costs $1500 was sold for $1200?

$1500 - $1200 = $300 = \frac{300}{1500} = \frac{1}{5} = .20 = 20\%$

QUIZ (O):
What simple annual rate of interest was paid if $5000 earned $300 interest in 2 years?
Averages / Means:
An average or mean is the sum of a series of numbers divided by the count (the number of numbers).

Example:
The student test scores average
\[ 92 + 88 + 72 + 80 + 86 = 418 \]
Divide 418 by 5 (number of test scores)
\[ 418 \div 5 = 83.6 \]

Quiz (P):
What is the average temperature in Albany, NY during the 6 selected dates? The temperatures were as follows: 32, 18, 21, 15, 34, 24

Medians:
A median is the middle number of a distribution where 50% of the numbers are greater and 50% of the numbers are less than the median. Remember to always list the number set in increasing numerical order first. If you have an even number set, the median is the average of the two middle numbers.

Example:

What is the median of the number grouping: 10, 40, 32, 18, 6, 2, 7?
Place the numbers in order:

\[
\begin{align*}
2 & \\
6 & \\
7 & \uparrow \quad 50\% \\
\hline \\
10 & \\
18 & \\
32 & \downarrow \quad 50\% \\
40 & 
\end{align*}
\]

10 is the correct median number

Quiz (Q):
The following temperatures were recorded at the airport: 32, 18, 21, 15, 34, 24. What is the median temperature?
Complex Problems

The PEF examination does contain complex word problems. They often have more written words and figures than are needed to solve the problem. While these questions can be a challenge, using a logical approach and accurate calculations will simplify the solution.

In this section we have subdivided the typical categories of problems and noted typical usage and formulas and/or calculation strategies. The examples will guide you through the mental thought process of solving the problem.

Additional practice questions are provided in a supplement to this guide.
Contents

Ratios
   Percentages
   Proportions

Distance / Velocity

Space Relationships

Work Problems

Counting / Segregating Data
**Ratios:**

Ratios are relationships between numbers that are found in many of the arithmetic problems. A ratio is a comparison between two quantities and compares a part to a part (as opposed to fractions that compare a part to the whole). A ratio can be expressed as a fraction, a number to a number or with the colon (:) symbol. For example, 3/5, 3 to 5 or 3:5. Typically a description is included to describe the class of ratio.

Samples:

To make a cake you need 1 cup of flour and 3 eggs. The ratio of flour to eggs is 1:3

To make a banana split you need 1 banana, 3 scoops of ice cream, 2 toppings and 1 serving of whipped cream. The ratio of bananas to ice cream to toppings to whipped cream is 1:3:2:1. There are seven total parts.

Remember that order matters in a ratio, so the answer should be in the same order that the question asked for. Also, pay close attention to the output units the question is asking for. Sample:

There are 4 red jelly beans in a jar and 7 blue jelly beans. What is the ratio of blue to red jelly beans? Answer = The ratio of blue to red jelly beans is 7:4

**Example Question (A)**

The ratio of red to white paint in a gallon of pink paint is 4:1. You need 20 gallons of pink paint to cover your kitchen. How many gallons of white and red paint do you need?

**Solution Thinking Steps:**

1. Determine what outcome is being asked for (what units)
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.
Example Question (B)
Each costume for a Broadway show takes 3 parts sequins, 5 parts lace and 7 parts spandex. If they have 300 costumes, how many parts of lace did they use?

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.

Percentages:
Percentages are a subset of ratios that are used when the numerator and denominator are the same units. It is designated by the % sign. For example, 26% can also be converted to a decimal by moving the decimal two spaces to the left and removing the '%' sign:

\[
26\% = .26 \\
5\% = .05 \\
100\% = 1.00
\]

This way the number can be used in a calculation (i.e. multiplication) easily.

Percentages are really fractions that are describing a part in terms of the whole.

Some examples of percentages are:

Agency: \[
\frac{\text{# calls returned from complaints}}{\text{total number complaints}}\]

This tells you what percentage of complaints get a return call.

Bank: \[
\frac{\text{# transactions at drive in window}}{\text{total bank transactions}}\]

This tells you what percentage of bank transactions are from the drive through windows.

Example Question (C)
Tom, John and Mary ate lunch together. Tom's meal cost 50% more than John's and Tom's cost 75% more than Mary's. John paid $10 for his meal. What is the total that the three paid for lunch?

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Determine what order you need to do each calculation in.
4. Calculate and label the answer in the units/order asked for.
Example Question (D)
140 people go to a pizza place each day. Assuming the place is open 7 days a week, it receives 140 customers each day and it has 40% of its customers eat in, what is the number of customers that order take out in a week?

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.

Proportions:
A proportion is a relationship between two ratios of equal value. For example, a:b = c:d.

If you were told that for every four people that come to your party, you need 3 bottles of soda, then the ratio of people to soda is 4:3. Then say you knew that 16 people are coming to your party. How many bottles of soda do you need?

You need to calculate the proportion. 4:3 = 16:?

A simple way to do this is to cross multiply. If set up as fractions, 4/3 x 16/X. Cross multiply the numerator of the 1st fraction with the denominator of the second and vice versa. So, 4 x X = 3 x 16; 4X = 48.

Now simply solve for X (4 times what number = 48). X = 12. The missing number in the ratio is 12. Then write the answer in proportion form 4:3 = 16:12. You will need to buy 12 bottles of soda if 16 people are at you party.

You may also see the proportion sign written as a double colon (::). So the answer could also be written as 4:3 :: 16:12 (4:3 is proportionate to 16:12).

An alternate method is to use the product of the means is equal to the product of the extremes. When the proportion is written as 4:3 :: 16:?, the extremes are the outside numbers (4 and ?) and the means are the inside numbers (3 and 16). Simplified, 4X = 3 x 16, then solve for X.

Example Question (E)
A great party requires a mixture of happy people and games played in a ratio of 2:5. If there are 30 happy people who come to Sam’s party, how many games does he need to be played to make it a great party?

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.
Example Question (F)

It takes Martha 4 hours to travel the 10 blocks from her house to her work. How long will it take her (travelling at the same rate per block) to get from her house to the gym which is 35 blocks away?

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.

Distance / Velocity:
Distance and velocity problems are often the easiest problems to solve because of the simplicity of the formula: Distance = Rate x Time (D=R x T).

Often it is helpful to visualize the route by drawing a line and placing the given information on it before you do the calculations.

Example Question (G)
A train travels at an average speed of 45 miles per hour through suburban areas for 30 minutes then is able to travel at 65 miles per hour along the thruway for 2.5 hours. How far did the train travel in the 3 hours?

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.
Example Question (H)

A car traveled 60% of the way from town A to town B at an average speed of 50 mph. The car travels at an average speed of "S" mph for the remaining part of the trip. The average speed for the entire trip was 45 mph. What is "S"?

<table>
<thead>
<tr>
<th>Town</th>
<th>60%</th>
<th>40%</th>
<th>Town</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50mph</td>
<td>S mph</td>
<td>B</td>
</tr>
</tbody>
</table>

\[ R = 45 \text{mph} \]

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.

Space Relationships:
These problems are focused on issues of area, perimeter and volume. Often basic arithmetic formulas are sufficient to make these simple calculations. The problem may also be made more complicated by adding extraneous information or making the diagram more complex. Breaking the problem into segments simplifies the calculations. The formulas that are typically used are:

<table>
<thead>
<tr>
<th>Area of a rectangle or square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area = the # of square units that can be put into the region enclosed by the sides</td>
</tr>
<tr>
<td>Area = width x length or area = height x base</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Perimeter of rectangle or square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter = The sum of the length of all sides</td>
</tr>
<tr>
<td>[ P = \text{width side 1} + \text{length side 2} + \text{width side 3} + \text{length side 4} \text{ or } P = (2 \times \text{width}) + (2 \times \text{length}) ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume = the # of cubic units that can be put into the region enclosed by the sides</td>
</tr>
<tr>
<td>[ V = \text{length} \times \text{width} \times \text{height} ]</td>
</tr>
</tbody>
</table>

Common conversions:

<table>
<thead>
<tr>
<th>Length</th>
<th>1 foot = 12 inches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 yard = 3 feet</td>
</tr>
<tr>
<td>Area</td>
<td>1 sq ft = 144 sq inches</td>
</tr>
<tr>
<td></td>
<td>1 sq yd = 9 sq ft</td>
</tr>
<tr>
<td>Volume</td>
<td>1 quart = 2 pints</td>
</tr>
<tr>
<td></td>
<td>1 gal = 4 quarts</td>
</tr>
<tr>
<td>Example Question (I)</td>
<td>Solution Thinking Steps:</td>
</tr>
<tr>
<td>----------------------</td>
<td>--------------------------</td>
</tr>
</tbody>
</table>
| If the length of a rectangle is increased by 25% and the width is decreased by 20% then what is the change in area? | 1. Determine what outcome is being asked for (what units).  
   2. Break the problem down into the parts needed to calculate.  
   3. Calculate and label the answer in the units/order asked for. |

<table>
<thead>
<tr>
<th>Example Question (J)</th>
<th>Solution Thinking Steps:</th>
</tr>
</thead>
</table>
| A contractor is calculating the flooring sq footage on a 3 story tower. The floors are a regular rectangle shape with 14’ length and a 6’ width. What is the square footage? | 1. Determine what outcome is being asked for (what units).  
   2. Break the problem down into the parts needed to calculate.  
   3. Calculate and label the answer in the units/order asked for. |
Example Question (K)

Water has been poured into an empty rectangular tank at the rate of 6 cubic feet per minute for 5 minutes. The length of the tank is 5 feet and the width is one half of the length. How deep is the water in the tank?

Solution Thinking Steps:

1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.

Work Problems:

Work issues are so varied that oftentimes these questions address generic types of issues. Some of these deal with salary, time, fringe benefits, etc. As with the other problems watch for critical clues and eliminate extraneous information. The calculations will involve basic arithmetic that you have previously learned and used.

Remember to use the solution thinking steps:

1. Determine what outcome is being asked for (and what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.
Example Question (L)
If in 2004, 2005, and 2006 a worker received 4% more in salary each year (a 4% raise each year) then she did in the previous year, how much more did she receive in 2006 than 2004?

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.

Example Question (M)
The assessed value of a house is $75,000. The assessed value is 60% of the market value of the house. If taxes are $3.00 for every $1000 of the market value of the house how much are the taxes on the house?

Assessed Value = $75,000
= 60% of the market value
Taxes are $3 for every $1000 market value

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.

Counting / Segregating Data:
Counting and segregating data involves understanding relationships and outlining separate classes of information. These problems involve making visual diagrams of the data to separate the information. Again, remember to break the problem down into simple calculations. These problems become easy once you simplify!

Use the Solution Thinking Steps:
1. Determine what outcome is being asked for (and what units).
2. Break the problem down into the parts needed to calculate.
3. Calculate and label the answer in the units/order asked for.

Example Question (N)
In a group of people solicited by area charities, 25% contributed $50, 45% contributed $25 and the rest contributed $10. If the charity received a total of $600

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
2. Break the problem down into the
<table>
<thead>
<tr>
<th>Question</th>
<th>Solution Thinking Steps</th>
</tr>
</thead>
</table>
| Example Question (O) | On a promotional exam a woman scored a 143 and a man scored 130 on a scale of 0 - 160. What is the ratio of scores based on a scale of 0 - 100?  

1. Determine what outcome is being asked for (what units).  
2. Break the problem down into the parts needed to calculate.  
3. Calculate and label the answer in the units/order asked for. |
| Example Question (P) | In a certain town 40% of the people have blond hair and 25% have brown eyes. Of these, 10% have both blonde hair and brown eyes. What percentage of the people have neither blonde hair nor brown eyes?  

1. Determine what outcome is being asked for (what units).  
2. Break the problem down into the parts needed to calculate.  
3. Calculate and label the answer in the units/order asked for. |
| Example Question (Q) | In a local pharmacy a pharmacist is adding three ingredients to make up a special prescription. The formula is to be put in a 16 oz bottle. The ratio of ingredients A, B, C is 1 : 2 : 7. How many ounces are needed of the second ingredient for the 16 oz bottle?  

1. Determine what outcome is being asked for (what units).  
2. Break the problem down into the parts needed to calculate.  
3. Calculate and label the answer in the units/order asked for. |
Answer Key

Arithmetic Calculations

<table>
<thead>
<tr>
<th>Quiz</th>
<th></th>
<th>Quiz</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>Quiz I</td>
<td>6541/10,000</td>
</tr>
<tr>
<td>B</td>
<td>30</td>
<td>Quiz J</td>
<td>$1149.22</td>
</tr>
<tr>
<td>C</td>
<td>132</td>
<td>Quiz K</td>
<td>$919.38</td>
</tr>
<tr>
<td>D</td>
<td>29/7</td>
<td>Quiz L</td>
<td>$29.10</td>
</tr>
<tr>
<td>E</td>
<td>1 1/8”</td>
<td>Quiz M</td>
<td>12</td>
</tr>
<tr>
<td>F</td>
<td>$60</td>
<td>Quiz N</td>
<td>11,200</td>
</tr>
<tr>
<td>G</td>
<td>6</td>
<td>Quiz O</td>
<td>3%</td>
</tr>
<tr>
<td>H</td>
<td>3/5</td>
<td>Quiz P</td>
<td>24 degrees</td>
</tr>
<tr>
<td>Q</td>
<td>22.5 degrees</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Complex Problems

<table>
<thead>
<tr>
<th>Ratios</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Question A</td>
<td>16 gallons of red and 4 of white</td>
</tr>
<tr>
<td>Question B</td>
<td>1500 parts of lace</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Percentages</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Question C</td>
<td>$36.25</td>
</tr>
<tr>
<td>Question D</td>
<td>588 customers</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Proportions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Question E</td>
<td>75 games</td>
</tr>
<tr>
<td>Question F</td>
<td>14 hours</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distance / Velocity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Question G</td>
<td>185 miles</td>
</tr>
<tr>
<td>Question H</td>
<td>33 1/3 mph</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Space Relationships</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Question I</td>
<td>No change</td>
</tr>
<tr>
<td>Question J</td>
<td>252 sq ft</td>
</tr>
<tr>
<td>Question K</td>
<td>2.4 ft</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Work Problems</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Question L</td>
<td>8.2%</td>
</tr>
<tr>
<td>Question M</td>
<td>$375.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Counting/Segregating Data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Question N</td>
<td>$5350.00</td>
</tr>
<tr>
<td>Question O</td>
<td>89:81</td>
</tr>
<tr>
<td>Question P</td>
<td>45%</td>
</tr>
<tr>
<td>Question Q</td>
<td>3.2 oz</td>
</tr>
</tbody>
</table>
Arithmetic Calculations – Explanations

QUIZ (A):
A group of managers at a conference sit in rows with the same number of managers in each row. Each row must contain 2 managers and there must be at least 2 rows. A row is parallel to the front of the room. How many different arrangements are possible?

$45 = 3 \times 15$

$= 3 \times 3 \times 5$

Possibilities are:

$3 \times 15 \quad 9 \times 5$

$5 \times 9 \quad 15 \times 3$

The answer is: 4

QUIZ (B):
Find LCM of 5, 6 and 15:
Multiples of 5 are 10, 15, 20, 25, 30, 35, 40,...
Multiples of 6 are 12, 18, 24, 30, 36, 42, 48,...
Multiples of 15 are 30, 45, 60, 75, 90,...

When you look at the list of multiples, you can see that 30 is the smallest number that appears in each list. 30 can be divided without remainders by 5 (30/5=6), 6 (6/30=5) and 15 (30/15=2).
Therefore, the least common multiple of 5, 6 and 15 is 30.

The answer is: 30

QUIZ (C):
Find LCM of 11 and 12.
11 and 12 have no common multiples until you multiply them together.

$11 \times 12 = 132$

$12 \times 11 = 132$

Therefore, the LCM is their product.

The answer is: 132

QUIZ (D):
Write $4 \frac{1}{7}$ as a fraction:

Method 1:

$4 \times 7 + 1 = 29$

29 is the calculated numerator and 7 is the original denominator.

The answer is: 29/7

QUIZ (E):
Add the measurements of $1/2" + 3/8" + 1/4"$

Convert all fractions to lowest common denominator:

$1/2" = 4/8$

$3/8" = 3/8$

$1/4" = 2/8$

Add the numerators:

$4/8" + 3/8" + 2/8" = 9/8"$ or $1 \frac{1}{8}$

The answer is: 9/8 or 1 1/8

QUIZ (F):
Our team saved 1/4 of $240 raised in the fundraiser. What is the amount saved?
Remember that $240= \frac{240}{1}$

$\frac{240}{1} \times \frac{1}{4} = \frac{240}{4} = \frac{60}{1} (240 \times 1 \text{ and } 1 \times 4)$

The answer is: $60$
QUIZ (G):
A supervisor can produce a report in 2/3 of an hour. How many reports can he produce in 4 hours?

\[
\frac{4}{1} \div \frac{2}{3} = \\
\frac{4}{1} \times \frac{3}{2} = \frac{12}{2} = 6
\]

The answer is: 6 reports

QUIZ (H):
It takes 1 1/4 hours to get from Syracuse to Utica traveling at a constant rate of speed. What part of the distance is traveled in 3/4 of an hour?

\[
\frac{3}{4} \div \frac{1}{4} = \left(\frac{3}{4}\right) / \left(\frac{5}{4}\right) \\
\frac{3}{4} \div \frac{5}{4} \\
\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}
\]

The answer is: 3/5

QUIZ (I):
Find the fraction that number .6541 represents.

4 places means denominator of 10,000

The answer is: 6541/10,000

QUIZ (J):
In collecting change for a fundraising drive the 3 locations have $100.27, $48.90 and $1000.05 respectively. How much has been collected?

$  100.27 \\
$    48.90 \\
$1000.05 \\
$1149.22

The answer is: $1149.22

QUIZ (K):
Of the $1149.22 collected 20% or 1/5 or .2 is to be allocated for expenses. How much remains in the account after expenses are taken?

Calculate $1149.22 \times \frac{1}{5} = $229.84

$1149.22 \\
- $229.84 \\
$919.38

The answer is: $919.38

QUIZ (L):
If travel is reimbursed at .485 cents per mile, what is the cost for 60 miles?

\[
\begin{align*}
60.00 \\
\times 0.485 \\
29.1
\end{align*}
\]

The answer is: $29.10
QUIZ (M):
If apples cost $.42 each how many apples can you buy for $5.04?

\[
\frac{5.04}{.42} = \frac{504}{42} \text{ (move the decimal point two places in each number)} = 12
\]
The answer is: 12

QUIZ (N):
The population of government workers in the central region was 10,000 in 2000. It is projected to increase by 12% by 2015. What is the projected population?

\[
10,000 \times 1.12 = 11,200
\]
The answer is: 11,200

QUIZ (O):
What simple annual rate of interest was paid if $5000 earned $300 interest in 2 years?

Interest in one year = $300 ÷ 2 = $150.

\[
\frac{150}{5000} = .03 \text{ or 3%}
\]
The answer is: 3% is annual rate

QUIZ (P):
What is the average temperature in Albany, NY during the 6 selected dates? The temperatures were as follows: 32, 18, 21, 15, 34, 24

\[
\frac{32 + 18 + 21 + 15 + 34 + 24}{6} = 24
\]
The answer is: 24 degrees

Quiz (Q):
The following temperatures were recorded at the airport: 32, 18, 21, 15, 34, 24. What is the median temperature?

Arrange the numbers in order:
15, 18, 21, 24, 32, 34

With even numbers take average of the middle two temperatures:
\[
\frac{21+24}{2} = \frac{45}{2} = 22.5
\]
The answer is: 22.5 degrees
Complex Problems – Explanations

Example Question (A)
The ratio of red to white paint in a gallon of pink paint is 4:1. You need 20 gallons of pink paint to cover your kitchen. How many gallons of white and red paint do you need?

Solution Thinking Steps:
1. Determine what outcome is being asked for (what units).
   = The total gallons of each type of paint

2. Break the problem down into the parts needed to calculate.
   = There are 5 total parts in a whole gallon of pink paint (4 red + 1 white)
   4 groups of this ratio will fit into your whole (20 gallons needed ÷ 5 parts in the whole = 4)
   Multiply your original ratio by the number of groups you need. 4/1 x 4/4 = 16/4

3. Label the answer in the units/order asked for.
   The answer is: You need 16 gallons of red paint and 4 gallons of white paint.

Example Question (B)
Each costume for a Broadway show takes 3 parts sequins, 5 parts lace and 7 parts spandex. If they have 300 costumes, how many parts of lace did they use?

Solution Thinking Steps:
1. Your outcome is:
   = number of lace parts used.

2. The problem ratio will be: lace parts per costume multiplied by the total costumes.

3. Remember the ratio is 3:5:7 and the total parts in one costume is 15 (this is 3+5+7). You need 300 costumes.

   There are 5 parts of lace in each costume.
   5 x 300 = 1500 parts of lace

   Alternatively, you could tell that one costume is 5/15 = 1/3 lace. You could then calculate that you need 4500 total parts to make 300 costumes (15 x 300). Then simply multiply the number of costumes in total by the fraction that is lace. 4500 x 1/3 = 4500/3 = 1500 parts of lace

   The answer is: 1500 parts of lace
Example Question (C)

Tom, John and Mary ate lunch together. Tom's meal cost 50% more than John's and Tom's cost 75% more than Mary's. John paid $10 for his meal. What is the total that the three paid for lunch?

Solution Thinking Steps:
1. Your outcome is Tom’s $ + John's $ + Mary's $ = $ Lunch Cost
2. Now with the given fact that John’s lunch cost $10.00, you can calculate Tom’s lunch: $10 + ($10 x .50) = $15
3. Then using Tom's $15.00 you can calculate Mary's lunch cost: .75 x $15 = $11.25
4. The total is: $15.00 + $10.00 + $11.25 = $36.25

The answer is: $36.25

Example Question (D)

140 people go to a pizza place each day. Assuming the place is open 7 days a week, it receives 140 customers each day and it has 40% of its customers eat in, what is the number of customers that order take out in a week?

Solution Thinking Steps:
1. Your outcome is the number of customers taking out per week.
2. You need to calculate the number of customers eating in and then multiply this by the number of days in a week.

60% take out each day (100-40)
140 customers each day
7 days in a week

3. Calculate and label the solution.

.6 x 140 = 84
84 x 7 = 588

Alternatively, you can calculate the number of customers/week and then multiply by the % take out.

7 x 140 = 980
980 x .6 = 588

The answer is: 588 customers
<table>
<thead>
<tr>
<th>Example Question (E)</th>
<th>Solution Thinking Steps:</th>
</tr>
</thead>
</table>
| A great party requires a mixture of happy people and games played in a ratio of 2:5. If there are 30 happy people who come to Sam’s party, how many games need to be played to make it a great party? | 1. Determine what outcome is being asked for (what units).  
   The number of games played.  
2. Break the problem down into the parts needed to calculate.  
   This is a proportion question. Given the happy people : games played ratio of 2:5, calculate the games needed to be played (X) when the happy people = 30.  
   \[ \frac{2}{5} :: \frac{30}{X} \]  
3. Calculate and label the answer in the units/order asked for.  
   You can cross multiply:  
   \[ \frac{2}{5} = \frac{30}{X}; 2X=150; X=75 \]  
   OR you can do the product of the means = the product of the extremes:  
   \[ 2 \times X = 5 \times 30; 2X=150; X=75 \]  
   The answer is: 75 games are needed to be played. |
Example Question (F)

It takes Martha 4 hours to travel the 10 blocks from her house to her work. How long will it take her (travelling at the same rate per block) to get from her house to the gym which is 35 blocks away?

Solution Thinking Steps:

1. Determine what outcome is being asked for (what units).

   How long it will take her to get from her house to the gym in hours?

2. Break the problem down into the parts needed to calculate.

   This is a proportion problem where the ratio of hours to blocks is 4:10. We are finding the number of hours (X) to travel 35 blocks.

   \[ \frac{4}{10} :: \frac{X}{35} \]

3. Calculate and label the answer in the units/order asked for.

   You can cross multiply:
   \[ \frac{4}{10} = \frac{X}{35}; \quad 10X=140; \quad X=14 \]

   OR you can do the product of the means = the product of the extremes:
   \[ 10 \times X = 4 \times 35; \quad 10X=140; \quad X=14 \]

   The answer is: It will take Martha 14 hours to travel the 35 blocks from her house to the gym.
Example Question (G)

A train travels at an average speed of 45 miles per hour through suburban areas for 30 minutes then is able to travel at 65 miles per hour along the thruway for 2.5 hours. How far did the train travel in the 3 hours?

DIAGRAM:

<table>
<thead>
<tr>
<th>Suburban Leg</th>
<th>Thruway Leg</th>
</tr>
</thead>
<tbody>
<tr>
<td>45mph x .5hour</td>
<td>65mph x 2.5hour</td>
</tr>
</tbody>
</table>

Solution Thinking Steps:

1. Determine what outcome is being asked for (what units).
   
   Your outcome is distance so the formula will look like:
   
   \[ D = R \times T \]

2. Break the problem down into the parts needed to calculate.
   
   Because of the different rates and times you can break up the equation:

   \[
   \text{Suburban Leg} \quad \text{Thruway Leg}
   
   \begin{align*}
   D &= 45 \text{mph} \times 0.5 \\
   D &= 65 \text{mph} \times 2.5
   \end{align*}
   
   Remember to keep values the same and convert hours to minutes or minutes to hours.

3. Calculate and label the answer in the units/order asked for.

   \[
   D = (45 \times 0.5) + (65 \times 2.5)
   
   D = 22.5 + 162.5
   
   D = 185 \text{ miles}
   
   The answer is: 185 miles

Example Question (H)

A car traveled 60% of the way from town A to town B at an average speed of 50 mph. The car travels at an average speed of "S" mph for the remaining part of the trip. The average speed for the entire trip was 45 mph. What is "S"?

<table>
<thead>
<tr>
<th>Town</th>
<th>60%</th>
<th>40%</th>
<th>Town</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>S</td>
<td>B</td>
</tr>
</tbody>
</table>

R = 45mph

Solution Thinking Steps:

1. Determine what outcome is being asked for (what units).
   
   Your outcome is to find "S" which is the average speed in mph for the trip.

2. Break the problem down into the parts needed to calculate.
   
   As we do not know distance we can use 100 (or actually any number as the distance does not make any difference in this calculation.
3. Calculate and label the answer in the units/order asked for.

\[ D = R \times T \]

1st leg --- \(.60(100) = 50\text{mph} \times T\)

\[ 60 = 50 \times T \]

\[ T = \frac{60}{50} \]

2nd leg --- \(.40(100) = S \text{mph} \times T\)

\[ 40 = S \times T \]

\[ T = \frac{40}{S} \]

\[ \frac{60}{50} = \frac{40}{S}; \quad 60S = 2000; \quad S = 33 \frac{1}{3} \text{ mph} \]

The answer is: \(33 \frac{1}{3} \text{ mph}\)

<table>
<thead>
<tr>
<th>Example Question (I)</th>
<th>Solution Thinking Steps:</th>
</tr>
</thead>
<tbody>
<tr>
<td>If the length of a rectangle is increased by 25% and the width is decreased by 20% then what is the change in area?</td>
<td>1. Determine what outcome is being asked for (what units).</td>
</tr>
<tr>
<td></td>
<td>Outcome is the area % change and an indication of an increase or decrease in area. Area = L \times W</td>
</tr>
<tr>
<td></td>
<td>2. Break the problem down into the parts needed to calculate.</td>
</tr>
<tr>
<td></td>
<td>For the length we are adding .25 and for the width we are subtracting .20</td>
</tr>
<tr>
<td></td>
<td>3. Calculate and label the answer in the units/order asked for.</td>
</tr>
</tbody>
</table>
|                       | \[ (1.00L + .25L) \times (1.00W - .20W) \]
|                       | \[ = 1.25L \times .80W \]
|                       | \[ = 1.25 \times .80 \]
|                       | \[ = 1 \]
|                       | The new area is the same as the old area. |
|                       | The answer is: There is no change. |
Example Question (J)

A contractor is calculating the flooring square footage on a 3 story tower. The floors are a regular rectangle shape with 14’ length and a 6’ width. What is the square footage?

Solution Thinking Steps:

1. Determine what outcome is being asked for (what units).
   Area in square feet.

2. Break the problem down into the parts needed to calculate.
   \[ A = L \times W \]  Remember there are three stories.

3. Calculate and label the answer in the units/order asked for.
   \[ A = 14 \times 6 \]
   \[ A = 84 \]
   \[ 84 + 84 + 84 \text{ (or } 84 \times 3) = 252 \text{ square feet} \]
   The answer is: 252 square feet.

Example Question (K)

Water has been poured into an empty rectangular tank at the rate of 6 cubic feet per minute for 5 minutes. The length of the tank is 5 feet and the width is one half the length. How deep is the water in the tank?

Solution Thinking Steps:

1. Determine what outcome is being asked for (what units).
   The outcome is the number of feet of water depth (the height of the water).

2. Break the problem down into the parts needed to calculate.
   Calculate the amount of water by multiplying:
   \[ 6 \text{ cu ft/min} \times 5 \text{ min} = 30 \text{ cu ft} \]
   Calculate the height of the water by using the volume formula.

3. Calculate and label the answer in the units/order asked for.
Example Question (L)

If in 2004, 2005, and 2006 a worker received 4% more in salary each year (a 4% raise each year) then she did in the previous year, how much more did she receive in 2006 than 2004 (as a percent)?

<table>
<thead>
<tr>
<th>Year</th>
<th>Salary</th>
<th>Salary after 4% raise</th>
<th>Salary in 2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>$S</td>
<td>$S + ($S \times 0.04)</td>
<td>$S 2005 + ($S 2005 \times 0.04)</td>
</tr>
<tr>
<td>2005</td>
<td>$S 2005% + ($S 2005% \times 0.04)</td>
<td>$S 2005 + ($S 2005% \times 0.04)</td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>$S 2006% + ($S 2006% \times 0.04)</td>
<td>$S 2005 + ($S 2005% \times 0.04)</td>
<td></td>
</tr>
</tbody>
</table>

Solution Thinking Steps:

1. The outcome is the difference in dollars between the 2004 salary and the 2006 salary.
2. Convert 4% to .04 to do your multiplication. We need to calculate the 2005 salary and then the 2006 salary.
3. To determine the 2005 salary, let’s assume the 2004 salary ($S) is 1.

   \[ \text{Then the 2005 salary is } S + (S \times 0.04) = 1 + 0.04 = 1.04 \]

4. The 2006 salary is the $S 2005 + (S 2005 \times 0.04) = 1.04 + (1.04 \times 0.04) = 1.04 + 0.042 = 1.082$

5. Remember the difference between the 2004 and 2006 salaries is the $% \text{ increase/100 } = \text{ amount of increase/original value}$.

   The amount of the change is $1.082 - 1 = 0.082$, so: $\% \text{ increase/100 } = 0.082/1$

   Cross multiply to get: $\% \text{ increase } = 8.2\%$

You can check your work by simply placing in a $S$ salary amount and check the salary increase.

The answer is: 8.2% more money
Example Question (M)
The assessed value of a house is $75,000. The assessed value is 60% of the market value of the house. If taxes are $3.00 for every $1000 of the market value of the house how much are the taxes on the house?

Assessed Value = $75,000
= 60% of the market value
Taxes are $3 for every $1000 market value

Solution Thinking Steps:
1. The outcome is the $ amount of the taxes
2. First we find market value which is simply:
   \[ 75,000 = 60\% \times \text{market value} \]
   \[ 75,000 = 0.60 \times \text{market value} \]
   \[ \text{market value} = \frac{75,000}{0.6} \]
   \[ \text{market value} = 125,000 \]
3. Then based on $125,000 market value, calculate the taxes:
   Taxes are $3/$1000 market value which converts to .003
   \[ 125,000 \times 0.003 = 375.00 \]
   You can also calculate this as a proportion:
   $3:$1000 :: $X:$125,000; 1000X = 375,000; \[ X = 375 \]
   The answer is: $375

Example Question (N)
In a group of people solicited by area charities, 25% contributed $50, 45% contributed $25 and the rest contributed $10. If the charity received a total of $600 from the group contributing $10, how much was contributed by the total group?

<table>
<thead>
<tr>
<th>Contribution</th>
<th>% of Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50</td>
<td>25%</td>
</tr>
<tr>
<td>$25</td>
<td>45%</td>
</tr>
<tr>
<td>$10</td>
<td>? (30%)</td>
</tr>
</tbody>
</table>

Group of $10 contributors totaled $600

Solution Thinking Steps:
1. The outcome is the total dollar contribution of the group.
2. First the % of $10 contributors can be found by merely adding 25% + 45% = 70% and subtracting 70% from 100% equaling 30%
3. Then calculate the # of people (X) in the $10 group:
   \[ 600 = 10 \times X \text{ people}; 600/10 = X; \ X = 60 \]
4. If 30% of the group = 60 people then find the total number of people (X):
   \[ 60 = 0.30 \times X \text{ total people}; 60/0.3 = X; 200 = X \]
5. Then, calculate the subgroup populations and contributions:
   $50 contributors = (200 \times 25%) = 50 people;
   $50 per person = $50 \times 50 = $2500

   $25 contributors = (200 \times 45%) = 90 people;
   $25 per person = $25 \times 90 = $2250
Example Question (O)

On a promotional exam a woman scored 143 and a man scored 130 on a scale of 0 - 160. What is the ratio of the woman’s score to the man’s score based on a scale of 0 - 100?

Solution Thinking Steps:

1. The outcome is a proportion between the woman’s score and the man’s score.
2. A diagram may be helpful to visualize relationships
3. Convert the scores for both people from a scale of 160 to a scale of 100

Woman

143:160 :: X:100
160X = 14300
X = 89.375
X = 89 woman’s score out of 160

Woman

143:160 :: X:100
160X = 14300
X = 89.375
X = 89 woman’s score out of 160

Man

130/160 = X/100
160X = 13000
X = 81.25
X = 81 man’s score out of 100

The answer is: 89:81

Example Question (P)

In a certain town 40% of the people have blond hair and 25% have brown eyes. Of these, 10% have both blond hair and brown eyes. What percentage of the people have neither blond hair nor brown eyes?

Solution Thinking Steps:

1. The outcome is the # of people who do not have blond hair or brown eyes.
2. Diagram to segregate the information (see left)
3. From the diagram we can see that by adding 30% + 10% + 15% = 55% of the population falls into the categories mentioned.

Based on a population of 100% we can subtract
100% - 55% = 45%

45% of the population have neither blond hair nor brown eyes.

The answer is: 45%
Example Question (Q)

In a local pharmacy a pharmacist is adding three ingredients to make up a special prescription. The formula is to be put in a 16 oz bottle. The ratio of ingredients A, B, C is 1:2:7. How many ounces are needed of the second ingredient for the 16 oz bottle?

Solution Thinking Steps:

1. The outcome is the # of ounces of ingredient B

2. Set up a ratio to determine the number of ounces using the ratio given where X is the number of parts and solve for X. Remember to calculate the ratio equal to the whole you are looking for (16 oz).

   
   \[
   \begin{align*}
   (1X) + (2X) + (7X) &= 16 \\
   10X &= 16 \\
   X &= 1.6 \text{ oz}
   \end{align*}
   \]

3. Then multiply the value of X you solved for by the ratio of ingredients to determine how much of each ingredient is needed for a 16 oz bottle.

   
   \[
   \begin{align*}
   1X : 2X : 7X &= 16 \\
   1(1.6) : 2(1.6) : 7(1.6) \\
   1.6 : 3.2 : 11.2.
   \end{align*}
   \]

Therefore ingredient A = 1.6 oz, B = 3.2 oz and C = 11.2 oz. You can check this by making sure 1.6 + 3.2 + 11.2 = 16.

The answer is: 3.2 oz.
Glossary
Arithmetic Reasoning

The overarching purpose of the use of charts and graphs is to visually summarize raw data in a format that helps the reader understand the information in a context.

That is to show raw data in categories that help us understand comparison / contrast to variables or conditions that we could not easily understand in text.

This glossary is a brief synopsis or "cheat sheet" referring to some of the salient points of the guide.

### Rational Numbers:

<table>
<thead>
<tr>
<th>Rational Numbers</th>
<th>Integers</th>
</tr>
</thead>
<tbody>
<tr>
<td>The set of integers {-4, -3, -1, 0, 1, 2, 3, 4…} and set of fractions {a/b} or examples {1/2, 3/16…} are called rational numbers.</td>
<td>Positive Integers 0,1,2,3,4</td>
</tr>
<tr>
<td></td>
<td>Negative Integers -1,-2,-3,-4</td>
</tr>
<tr>
<td></td>
<td>(even numbers divisible by 2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculations</th>
<th>Least common multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product of Prime Factor</td>
<td></td>
</tr>
<tr>
<td>Multiple combinations of numbers multiplied have the same product.</td>
<td></td>
</tr>
<tr>
<td>24=2 x 12</td>
<td></td>
</tr>
<tr>
<td>=4 x 6</td>
<td></td>
</tr>
<tr>
<td>= 2 x 2 x 2 x 3</td>
<td></td>
</tr>
<tr>
<td>This is the smallest number that is a common multiple of both numbers.</td>
<td></td>
</tr>
<tr>
<td>The least common multiple of 11 and 12 is 132.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A fraction is two integers one on top of another that represent a portion of a whole (i.e., (\frac{1}{2}) of 1 is 50% of the whole). The top</td>
</tr>
</tbody>
</table>
number is the numerator (number of pieces) and the bottom number denominator (describes the number of pieces of the whole).

<table>
<thead>
<tr>
<th>Simple Interest</th>
<th>Simple Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple interest is based on interest during one year.</td>
<td>What annual rate of interest was paid if $5000 earned $300 interest in 2 years?</td>
</tr>
<tr>
<td></td>
<td>$150/5000 = .03 or 3%</td>
</tr>
<tr>
<td></td>
<td>3% is annual rate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compound Interest</th>
<th>Compound Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compound interest is calculated on multiple years where interest and principle is added at the end of one year and then multiplied by interest percentage, therefore adding more interest over multiple years.</td>
<td>What will $1000 be worth after 3 years if it earns rate of 5% compounded annually?</td>
</tr>
<tr>
<td></td>
<td>Year 1 interest is 1000 x 5% = $50</td>
</tr>
<tr>
<td></td>
<td>$1050 end of year one</td>
</tr>
<tr>
<td></td>
<td>Year 2 interest is 1050 x 5% = 1102.50</td>
</tr>
<tr>
<td></td>
<td>1102.50 end of year 2</td>
</tr>
<tr>
<td></td>
<td>Year 3 interest 1102.5 x 5% = 1157.63</td>
</tr>
<tr>
<td></td>
<td>1157.63 is compounded rate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean</th>
<th>Example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Mean is the sum of a series of numbers divided by the number of numbers</td>
<td>32+65+84+22+33 = 236</td>
</tr>
<tr>
<td></td>
<td>236÷5 = 47.2</td>
</tr>
</tbody>
</table>
**Complex Numbers:**

Complex problems often have more written words and figures than needed to solve the problem. While these are a challenge, using the logical approach and accurate calculations will simplify the solution.

<table>
<thead>
<tr>
<th>Averages</th>
<th>Example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add the values and divide by sum of the values.</td>
<td>Raw Data (3, 5, 2, 6, 1, 9, 1)</td>
</tr>
<tr>
<td></td>
<td>Solution:</td>
</tr>
<tr>
<td></td>
<td>1. Add</td>
</tr>
<tr>
<td></td>
<td>[3 + 5 + 2 + 6 + 1 + 9 + 1] [= \frac{28}{7}]</td>
</tr>
<tr>
<td></td>
<td>2. 28/7</td>
</tr>
<tr>
<td></td>
<td>3. 4.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Medians</th>
<th>Example:</th>
</tr>
</thead>
<tbody>
<tr>
<td>This measure divides a set of values up into two halves. This measure defines the 50th percentile of data.</td>
<td>Raw Data (3, 5, 2, 6, 2, 1, 9, 1, 7)</td>
</tr>
<tr>
<td></td>
<td>Solution:</td>
</tr>
<tr>
<td></td>
<td>1. Arrange in ascending order</td>
</tr>
<tr>
<td></td>
<td>1, 1, 2, 2, 3, 5, 6, 7, 9</td>
</tr>
<tr>
<td></td>
<td>2. Select the number in the middle</td>
</tr>
<tr>
<td></td>
<td>1, 1, 2, 2, 3, 5, 6, 7, 9</td>
</tr>
<tr>
<td></td>
<td>3. Median = 3</td>
</tr>
<tr>
<td></td>
<td>Example:</td>
</tr>
<tr>
<td></td>
<td>Raw Data (12, 15, 18, 19, 14, 16)</td>
</tr>
<tr>
<td></td>
<td>Solution:</td>
</tr>
<tr>
<td></td>
<td>1. Arrange in ascending order</td>
</tr>
<tr>
<td></td>
<td>12, 14, 15, 16, 18, 19</td>
</tr>
<tr>
<td></td>
<td>2. Select the two numbers in the middle and average them</td>
</tr>
<tr>
<td></td>
<td>[\frac{15 + 16}{2} = 15.5]</td>
</tr>
<tr>
<td></td>
<td>3. Median = 15.5</td>
</tr>
</tbody>
</table>
### Ratio / Percentage

<table>
<thead>
<tr>
<th><strong>Percentage:</strong></th>
<th>are ratios where numerator and denominator are the same values.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Example:</strong></td>
<td>If the values are $ money in dollars .20¢ of $1.00 = 20%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Proportions:</strong></th>
<th>a percentage becomes a proportion when the denominator restricts or limits the numerator in some way.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Example:</strong></td>
<td>Agency: #calls returned from complaints total number complaints</td>
</tr>
<tr>
<td></td>
<td>Bank: # transactions at drive window total bank transactions</td>
</tr>
</tbody>
</table>

### Distance / Velocity:

Distance and velocity problems are often the easiest problems to solve because of the simplicity of the formula: Distance = Rate x Time.

<table>
<thead>
<tr>
<th><strong>Example Question (A)</strong></th>
<th><strong>Solution Thinking Steps:</strong></th>
</tr>
</thead>
</table>
| A train travels at an average speed of 45 miles per hour through suburban areas for 30 minutes then is able to travel at 65 miles per hour along the thruway for 2.5 hours. How far did the train travel in 3 hours? | 1. Your outcome is distance so the formula will look like  
   \[ D = R \times T \]  
2. Because of the different rates and times you can break up the equation  
   Suburban Leg \[ 45 \text{mph} \times 0.5 \text{hr} \]  
   Thruway Leg \[ 65 \text{mph} \times 2.5 \text{hr} \]  
   (remember to keep values the same converting hours to minutes or minutes to hours)  
3. \[ D = (45 \times 0.5) \]  
   \[ D = 22.5 + 162.5 \]  
   \[ D = 184.5 \text{ miles} \] |

Diagram:

- Suburban Leg → Thruway Leg →
- 45mph x .5 hour → 65mph x 2.5 hour
Space Relationships:

These problems are focused on issues of area and perimeter as well as volume. Often the basics of arithmetic formulas are sufficient to make the simple calculations. The problem may be made more complicated by adding extraneous information or making the diagram more complex – to which breaking it into segments simplifies the effort. The formulas typically used are:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Area of a rectangle</strong></td>
<td>Area = width x length</td>
</tr>
<tr>
<td><strong>Perimeter of rectangle</strong></td>
<td>Per = (2 x width) + (2 x length)</td>
</tr>
<tr>
<td><strong>Volume</strong></td>
<td>Vol = length x width x height</td>
</tr>
</tbody>
</table>

Definitions (typical usage)

<table>
<thead>
<tr>
<th>Unit</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>1 foot = 12 inches</td>
</tr>
<tr>
<td></td>
<td>1 yard = 3 feet</td>
</tr>
<tr>
<td>Area</td>
<td>1 sq ft = 144 sq inches</td>
</tr>
<tr>
<td></td>
<td>1 sq yd = 9 sq ft</td>
</tr>
<tr>
<td>Volume</td>
<td>1 quart = 2 pints</td>
</tr>
<tr>
<td></td>
<td>1 gal = 4 quarts</td>
</tr>
</tbody>
</table>

Work Problems:

Work issues are so varied that often questions address generic types of issues. Some of these deal with salary, time, benefits, etc. As with the other problems watch for critical clues and eliminate extraneous information. The calculations will be normal arithmetic solutions that you have previously used.
**Example Question**

If in 2004, 2005, and 2006 a worker received 4% more in salary each year than she did in the previous year, how much more did she receive in 2006 than 2004?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>$S + .04s$</td>
</tr>
<tr>
<td>2004</td>
<td>2006</td>
</tr>
</tbody>
</table>

**Solution Thinking Steps:**

1. The outcome is difference in money between 2004 salary and 2006 salary.
2. Change 4% to .04 to do your multiplication.
3. The 2004 salary let’s call ‘S’ then 2005 salary is $S + .04s = 1.04s$.
4. The 2006 salary is the $1.04s \times .04$ which calculated is $4.16s$.
5. Therefore the difference between 2004 and 2006 salary is $4.16\%$.
6. You can check your work by simply placing in a $S$ salary amount and check the salary increase.

---

**Counting / Segregating Data:**

Counting and segregating data involves understanding relationships and outlining separate classes of information from others. These problems involve making a visual pictorial of data to separate the groupings of information.

**Example Question**

In a group of people solicited by area charities 25% contributed $50, 45% contributed $25 and the rest contributed $10. If the charity received a total of $600 from the group what dollar amount was contributed by each group?

<table>
<thead>
<tr>
<th>Contribution</th>
<th>% of Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50</td>
<td>25%</td>
</tr>
<tr>
<td>$25</td>
<td>45%</td>
</tr>
<tr>
<td>$10</td>
<td>? (30%)</td>
</tr>
</tbody>
</table>

Group of $10 contributors totaled $600.

**Solution Thinking Steps:**

1. The outcome is the total dollar contribution of the group.
2. First the % of $10 contributors can be found by merely adding 25% + 45% = 70% and subtracting 70% from 100% equaling 30%.
3. Then calculate # people in $10 group

\[
\frac{\text{Total amount}}{\text{Contribution amount}} = \text{Number of people}
\]

\[
\frac{600}{10} = 50 \text{ People}
\]

4. If 30% of the group = 50 people then to find the total number

\[
\frac{\text{Number of people}}{\text{Percentage}} = \text{Total number}
\]

\[
\frac{50}{0.3} = 200 \text{ People}
\]

5. The to calculate subgroup population contributions

\[
\text{Contribution amount} \times \text{Number of people}
\]

\[
50 \text{ contributors} \times (200 \times 25\%) = \text{(Your calculation)}
\]

\[
25 \text{ contributors} \times (200 \times 45\%) = \text{(Your calculation)}
\]
| $10 contributors x  = $600 |
| $50 \times 50 = 2500 |
| $25 \times 90 = $2250 |
| $600 |
| $2500 + $2250 + $600 + $5350 |
| 6. Total contribution is $5350. |